Half-life and Binding Energy

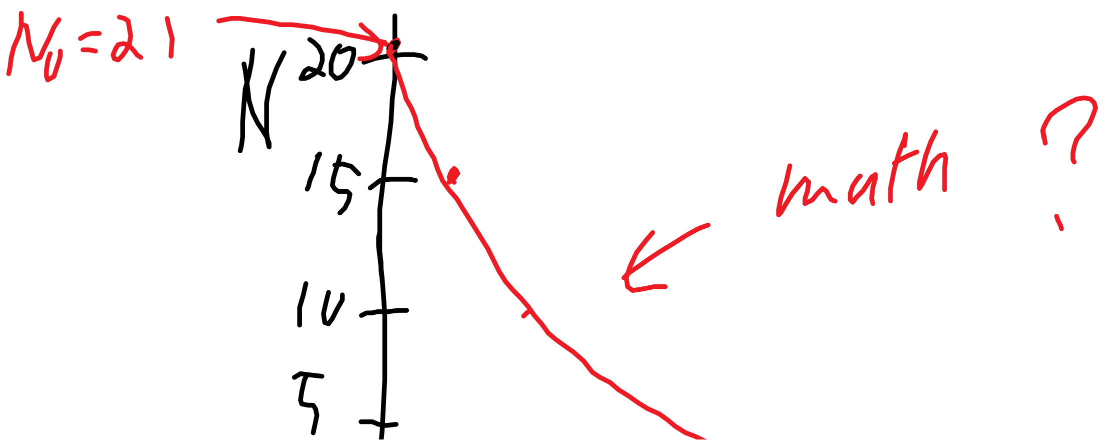
Hand in Lab and test corrections

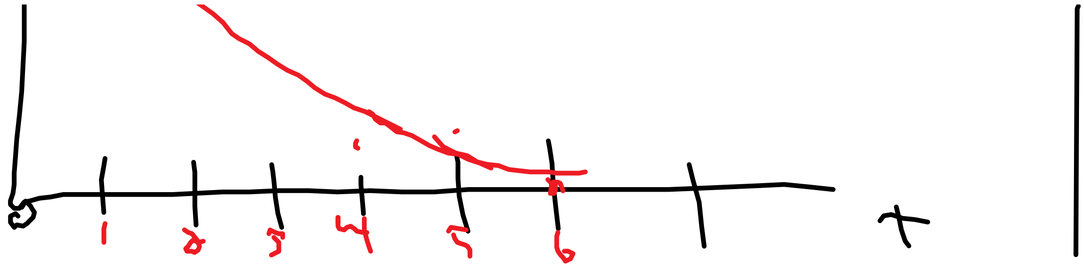
Get out 2 coins (borrow?)

Model Radioactive Half-life

Each Student is a radioactive nucleus. Nucleii decay randomly - model by flipping 2 coins, 2 heads=decay.

Number of undecayed nucleii (students) N	number of flips, t
21 = N_o	0
15	1
8	2
4	3
2	4
2	5
0	6





Half-life - time for half of the nucleii to decay
symbol = $t_{1/2}$
the half-life is about 2 flips for our model
if every half-life, half decay we can derive an equation:

$N = N_o (1/2)^{\#\text{ of half-lives}}$ $\#\text{half-lives} = t/t_{1/2}$

N is the number of undecayed nucleii, in number or in kg, or in activity, decays/s

N_o is the original amount of the radioactive sample.
t is time
$t_{1/2}$ is the half-life
p621 - carbon 14 decays with a half-life of 5730 years while polonium 194 decays in 0.7s

eg. A sample discovered at an archeological dig contains a fraction of the original carbon 14 found in nature. How old is the sample if
a) half the carbon 14 is left
b) it started with 4.0 micro grams μg and ended with 1.0 micro grams, μg.

c) if a sample started with 4.0 μg of carbon 14, how much will be left after 30 000 years?

p622 Q9-12
5 minutes -

a) half of the sample = 1 half-life so it is 5730years
b) N/N_o = 1.0μg/4.0μg = 1/4 = 2 half-lives
2x5730=11460 years
c) N_o = 4.0 μg t=30 000 years $t_{1/2}$=5730years
N=? $N=N_o(1/2)^{\#half\text{-}lives}$
$N = 4.0x(1/2)^{\wedge 30000/5730}$=0.11μg
30000/5730=5.2356
0.5^5.2356=0.0265
0.0265x4=0.106

Binding Energy (Chapter 31)

Where does the nuclear energy come from?

mass of a proton = 1.6605×10^{-27}kg.
An easier unit to work with is the atomic mass unit, u = 1/12 the mass of carbon 12
so the mass of a proton is 1.007825 u
the mass of neutron is 1.008665 u

Hey, but the isotopic mass of Helium is only 4.002603u.

Helium is 2 protons and 2 neutrons.
(2x1.007825)+(2x1.008665)=4.03298

the masses don't match. What's the deal?
the mass defect (missing mass) =
mass of protons and neutrons-isotopic mass
4.03298-4.002603=0.030377 u of mass
missing. Where did it go?

Einstein - Theory of relativity included an equation $E=mc^2$, that mass can change into energy or back.

1u of mass changed into energy =
$E=mc^2$
E is energy, in Joules or MeV
m is mass in kg or u
c is the speed of light, 2.9979×10^8 m/s
How much energy is created when 1 u =
1.6605×10^{-27}kg of mass is lost
E= 1.6605E-27x(2.9979E8)^2=
1.6605x(2.9979)^2=14.9236×10^{-11} J
MeV is a unit for energy for very small energies, = 1.602×10^{-13}J
so 14.9236×10^{-11}J(1MeV/1.602×10^{-13}~~J~~)
= 931.49MeV for every u of mass lost

So when 2 protons and 2 neutrons change into Helium, 0.030377 u of missing mass gets changed into energy.

the conversion factor = 931.49MeV/u
0.030377 u x 931.49MeV/u
0.030377x931.49=28.295872 MeV of energy is given off - this is what powers the stars - like our sun.

eg. What is the mass defect and binding energy (energy given off in the formation) of uranium 235,
isotopic mass of 235.043929918
atomic number 92
mass of a proton is 1.007825 u
the mass of neutron is 1.008665 u
931.49MeV for every u of mass lost

p642 Q1-4

Nuclear fusion is when small nucleii bond together to form a larger nucleus.
mass 92 protons= 1.007825x92=92.7199
mass of 235-92=143 neutrons =
1.008665x143=144.239095
mass defect = (92.7199+
144.239095)-235.043929918=1.915065082 u
binding energy = 931.49x1.915065082=
1,783.863973232 = 1783.9MeV

Half-life and Binding Energy

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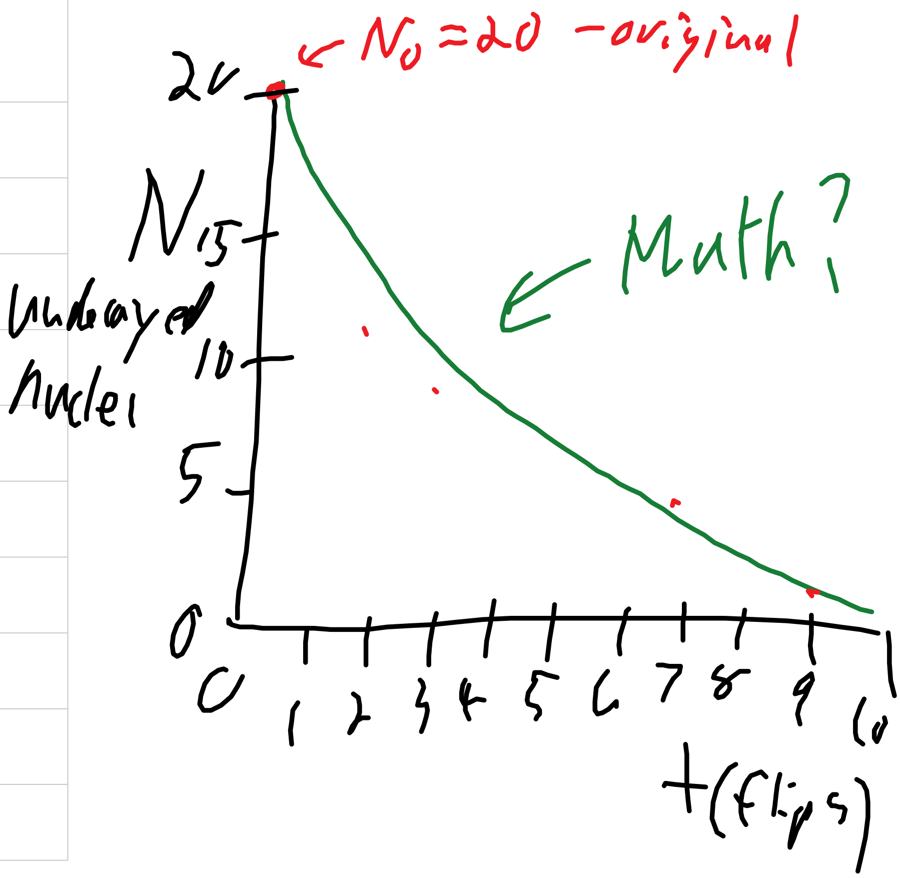
Model Radioactive Half-life

Each Student is a radioactive nucleus. Nucleii decay randomly - model by flipping 2 coins, 2 heads=decay.

Number of undecayed nucleii (students) N	number of flips, t
20 = N_o	0
15	1
11	2
8	3
8	4
6	5
5	6
4	7
3	8
1	9



Half-life is the time for half of the nuclei to decay, on average. symbol $t_{1/2}$

calculate the amount of undecayed nuclei, N, we can use the equation:

$N = N_o (1/2)^{\#\ of\ half\text{-}lives}$

N_o is the original amount, in g or number of atoms, or activity - decays/s

#of half-lives = $t/t_{1/2}$

eg p 621 table with half-life of carbon 14 is 5730 years while polonium 194 has a 0.7s half-life.

you have a sample of carbon 14.

a) what fraction of it is left after 5730 years?
   1/2
b) if you start with 4.0 μg and you end with 1.0 μg, how old is the sample?
   1/4=0.25 is 2 half-lives

N/N_o =1/4 so # of halflives = 2

2 x 5730=11460 years

a) if you start with 4.0 μg, how much is left after 30000 years?

$N = N_o (1/2)^{\#\ of\ half\text{-}lives}$

= 4.0x(1/2)^(30000/5730)=0.1062 = 0.11μg

# of halflives = $t/t_{1/2}$ = 30000/5730=5.2356

p622 Q9-12

Binding Energy (ch 31)

Mass of an isotope doesn't match the mass of protons and neutrons. What's the deal?

eg. mass of a proton = 1.6605 x 10^{-27}kg.
An easier unit to work with is the atomic mass unit, u = 1/12 the mass of carbon 12
so the mass of a proton is 1.007825 u
the mass of neutron is 1.008665 u

Hey, but the isotopic mass of Helium is only 4.002603u.

Helium is 2 protons and 2 neutrons.
(2x1.007825)+(2x1.008665)=4.03298

mass defect is the difference between the isotopic mass and the mass of protons and neutrons
4.03298-4.002603=0.030377 u
so Helium 4 has a mass defect of 0.030377 u

This mass that was lost in the formation of the atom, was given off as energy in nuclear fusion (small nuclei combining) in stars like our sun.

There is a conversion factor between mass and energy : 931.49 MeV/u

MeV is a unit for small energy, atomic level
MeV = 1.602×10^{-13}J

(we can prove that conversion factor using
$E=mc^2$ - Einstein's equation from relativity
1u = 1.6605×10^{-27}kg
c is the speed of light 2.9979×10^{8}m/s
$E = 1.6605 \times 10^{-27} \times (2.9979 \times 10^{8})^2$
1.6605x2.9979^2=14.9236 x10^{-11}J
14.9236/1.6022=9.3149 x 10^{2}
= 931.49 MeV for each u)

Ok so, if the mass defect of He 4 is
4.03298-4.002603=0.030377 u
then the binding energy =
mass defectxconversion factor
0.030377x931.49=28.295872
the binding energy = 28.296 MeV per helium4 nucleus.
eg. What is the mass defect and binding energy (energy given off in the formation) of
uranium 235,
isotopic mass of 235.043929918
atomic number 92
mass of a proton is 1.007825 u
the mass of neutron is 1.008665 u
931.49MeV for every u of mass lost
(92x1.007825)+(143x1.008665)=236.958995
236.958995-235.043929918 = 1.915065082
so the mass defect of U235 = 1.9151u

therefore, binding energy = mass defectx931.49MeV/u
1.915065082x931.49=1,783.863973232
binding energy = 1783.9MeV

p622 Q9-12 and p642 Q1-4

Half-life and Binding Energy

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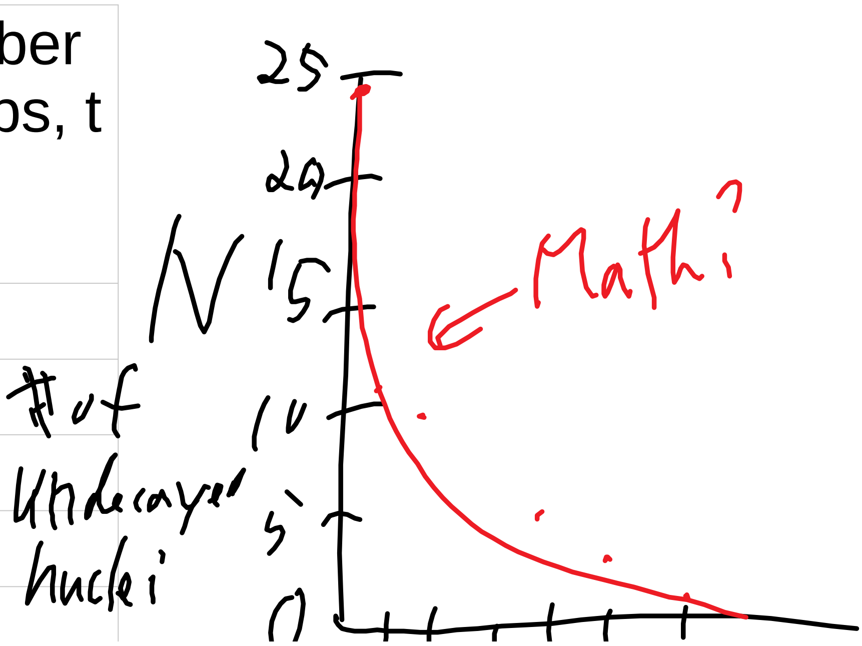
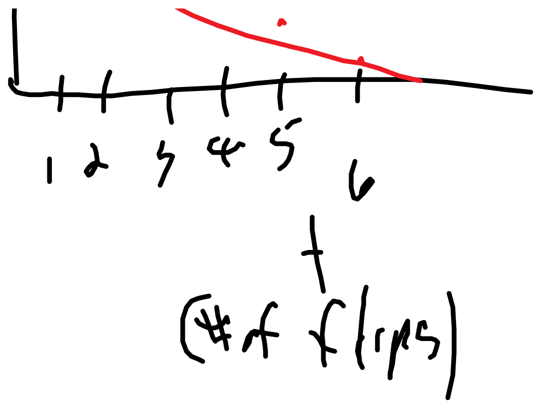
Model Radioactive Half-life

Each Student is a radioactive nucleus. Nucleii decay randomly - model by flipping 2 coins, 2 heads=decay.

Number of undecayed nucleii (students) N	number of flips, t
24= N_o	0
11	1
9	2
4	3
4	4



4	3
4	4
2	5
1	6



Half-life, $t_{1/2}$, is time for half of the radioactive nuclei to decay on average.

p621 table in the book
carbon 14 has a half-life of 5730 years
polonium 194 has $t_{1/2} = 0.7$ s

$N = N_o (1/2)^{\text{\# of half-lives}}$

N is the amount of radioactive sample left after time t. measured in number of nuclei, g, activity.
N_o is the original amount, at time t=0.
# of half-lives = time/half-life

eg. In an archeological dig you find a sample and measure the amount of carbon 14 and compare it to the normal natural occurrence.

a) how old is the sample if it is half the original amount of carbon 14?

5730years - one half-life

a) if you start with 4.0 μg and end with 1.0 μg, how old is the sample?

$(1/2)^2 = 1/4$ so 2 half-lives so 2x5730=11460 years

a) if you start with 4.0 μg, how much is left after 30000 years?

$N = N_o (1/2)^{\# \text{ of half-lives}}$

\# of half-lives = time/half-life

= 30000/5730=5.2356 half-lives

$N = 4.0\mu g\ (1/2)^{5.2356}$

= 4x(1/2)^5.2356=0.1062

0.11μg

p622 Q9-12

Binding Energy (ch31)

define the atomic mass unit,
u = 1/12 the mass of Carbon 12
but wait,
the mass of a proton is 1.007825u
the mass of a neutron is 1.008665u
6 protons and 6 neutrons don't add to 12, what's the deal with the missing mass - mass defect.
Einstein - theory of relativity proposed the relationship $E=mc^2$, mass m can change into energy, E by a factor of the speed of light, $c = 2.9979 \times 10^8$m/s

So, when protons combine to form bigger nuclei in fusion, some mass is lost and

becomes energy - powers the sun - stars.

conversion factor 931.49 MeV/u
if 1 u of mass changes into energy
931.49 MeV of energy are produced.
MeV is a unit for small energies
1 MeV = 1.6022 x 10^{-13}J

eg. What is the mass defect and binding energy of Helium 4? isotopic mass of 4.002603u recall m_p =1.007825u
m_n= 1.008665u
helium 4 has 2 protons and 2neutrons
(2x1.007825)+(2x1.008665)=4.03298
mass defect
= 4.03298-4.002603=0.030377 u
Binding energy = mass defect x conversion
0.030377x931.49=28.295872
28.296MeV of binding energy

what is the mass defect and binding energy for uranium 235?
atomic number 92
isotopic mass: 235.043929918u
p642 Q1-4
(92x1.007825)+(143x1.008665)
= 236.958995-235.043929918
=1.915065082 u mass defect
binding energy mass defect x conversion
1.915065082x931.49=1,783.863973232
1783.9 MeV of binding energy

1.915065082x931.49=1,783.863973232
1783.9 MeV of binding energy

where did the conversion factor come from?
$E=mc^2 = 1.6605\times10^{-27}kg \times (2.9979\times10^{8} m/s)^2$
$1.6605\times2.9979^2=14.9236 \times10^{-11} J$
$14.9236\times(1/1.6022)=9.3144 \times10^{2} MeV$
931.4MeV